REGLAS FUNDAMENTALES DE DIFERENCIACION

POR

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(Continuacion)

7. $y = \cos(a - bx) \therefore dy = -\text{sen}(a - bx)\, d(a - bx)$

$$= -\text{sen}(a - bx).\ -bdx = b\ \text{sen}(a - bx)\, dx.$$

8. $d \cos\sqrt{ax^2 + bx + c} = -\text{sen}\sqrt{ax^2 + bx + c}\, d(ax^2 + bx + c)^{\frac{1}{2}}$

$$= -\text{sen}\sqrt{ax^2 + bx + c}\ .\ \frac{2\,ax + b}{2\sqrt{ax^2 + bx + c}}\ dx$$

9. $d \cos nx \cos mx = \cos nxd \cos mx + \cos mxd \cos nx$

$$= -\cos nx\ \text{sen}\ mxdmx - \cos mx\ \text{sen}\ nxdnx$$

$$=-m\cos nx\,\mathrm{sen}\,mx-n\cos mx\,\mathrm{sen}\,nxdx.$$

270. $d\cos^n x=n\cos^{n-1}x\,d\cos x=-n\cos^{n-1}x\,\mathrm{sen}\,xdx$

1. $d\,\mathrm{sen}^m x\cos^n=x(\mathrm{sen}^m x.\ n\cos^{n-1}x.$

$$-\mathrm{sen}\,x+\cos^n x.\ m\,\mathrm{sen}^{m-1}x\cos x)\,dx$$

$$=(-\mathrm{sen}^{m+1}x\cos^{n-1}x+D\cos^{n+1}x\,\mathrm{sen}^{m-1}x)\,dx$$

2. $y=\mathrm{L}\cos x\therefore dy=\dfrac{d\cos x}{\cos x}=\dfrac{-\mathrm{sen}\,x}{\cos x}dx=-\mathrm{tg}\,xdx$

3. $y=(\cos x)^{\mathrm{sen}\,x}\therefore \mathrm{L}\,y=\mathrm{sen}\,x\,\mathrm{L}\cos x$

$$\frac{dy}{y}=\mathrm{sen}\,x.-\mathrm{tg}\,xdx+L\cos x.\cos xdx$$

$$dy=\cos x^{\,\mathrm{sen}\,x}\,(-\mathrm{sen}\,x\,\mathrm{tg}\,x+L\cos x^{\cos x})\,dx$$

4. $y=\cos(x^{\mathrm{sen}\,x})\therefore dy=-\mathrm{sen}\,(x^{\mathrm{sen}\,x})\,d\,(x^{\mathrm{sen}\,x}).$

Hacemos $u=x^{\mathrm{sen}\,x}$ y aplicamos L:

$$L\,u=\mathrm{sen}\,x\,L\,x\therefore\frac{du}{u}=\left(\frac{\mathrm{sen}\,x}{x}+\cos x\,L\,x\right)dx$$

$$du = x^{\text{sen}\,x}\left(\frac{\text{sen}x}{x} + L\,x^{\cos x}\right)dx$$

$$\therefore\ dy = -\text{sen}\,(x^{\text{sen}\,x})\,x^{\text{sen}\,x}\left(\frac{\text{sen}\,x}{x} + L\,x^{\cos x}\right)dx$$

275. $y = \cos\,\text{sen}\,(1-x)$. $dy = -\text{sen}\,\text{sen}\,(1-x)$

$$d\,\text{sen}\,(1-x) = -\text{sen}_2\,(1-x)\cos\,(1-x)\,d\,(1-x)$$

$$= \text{sen}_2\,(1-x)\cos\,(1-x)\,dx$$

Diferencial de la tangente.—Sea $y = \text{tg}\,u$.

$$dy = d\,\text{tg}\,u = d\,\frac{\text{sen}\,u}{\cos\,u} = \frac{\cos u\,d\,\text{sen}\,u - \text{sen}\,u\,d\cos u}{\cos^2 u}$$

$$= \frac{\cos^2 u + \text{sen}^2 u}{\cos^2 u}\,du = \frac{1}{\cos^2 u}\,du = \sec^2 u\,du$$

$$\therefore\ d\,(\text{tg}\,u) = \sec^2 u\,du \qquad \textbf{(14)}$$

La diferencial de la tangente es igual al cuadrado de la secante por la diferencial del arco.

Ejercicio 276 $y = \text{tg}\,(ax^2+bx+c)\ \therefore\ dy = \sec^2\,(ax^2+bx+c)$

$$d\,(ax^2+bx+c) = (2\,ax+bx)\sec^2\,(ax^2+bx+c)\,d\,x.$$

7. $d\,\text{tg}^n\,x^m = n\,\text{tg}^{n-1}\,x^m\,d\,\text{tg}\,x^m = n\,\text{tg}^{n-1}\,x^m\sec^2 x^m\,dx^m =$ $m\,n\,x^{m-1}\sec^2 x^m\,\text{tg}^{\,n-1}\,x^m\,dx.$

DIFERENCIAL DE LA COTANGENTE. $y = \cot u$

$$d \cot u = d\frac{1}{\operatorname{tg} u} = -\frac{d \operatorname{tg} u}{\operatorname{tg}^2 u} = -\frac{\sec^2 u}{\operatorname{tg}^2 u} = -\frac{1}{\cos^2 u}\frac{\cos^2 u}{\operatorname{sen}^2 u}$$

$$= -\frac{1}{\operatorname{sen}^2 u} \therefore d(\cot u) = -\operatorname{cosec}^2 u.$$

DIFERENCIAL DE LA SECANTE $y = \sec u$.

$$d \sec u = d\frac{1}{\cos u} = \frac{d \cos u}{\cos^2 u} = \frac{\operatorname{sen} u}{\cos^2 u} du = \operatorname{tg} u \sec u du$$

$$\therefore d(\sec u) = \sec u \operatorname{tg} u \, du.$$

DIFERENCIAL DE LA COSECANTE. $y = \operatorname{cosec} u$

$$d \operatorname{cosec} u = d\frac{1}{\operatorname{sen} u} = -\frac{d \operatorname{sen} u}{\operatorname{sen}^2 u} = -\frac{\cos u}{\operatorname{sen}^2 u} du$$

$$\therefore d(\operatorname{cosec} u) = -\cot u \operatorname{cosec} u \, du$$

DIFERENCIAL DEL SENO-VERSO. $y = \operatorname{vers} u$

$$d \operatorname{vers} u = d(1 - \cos u) = \operatorname{sen} u \, du$$

$$\therefore d(\operatorname{vers} u) = \operatorname{sen} u du.$$

DIFERENCIAL DEL COSENO VERSO. $y = \text{covers } u$

$$d \text{ covers } u = d\,(1 - \text{sen } u) = -\cos u du$$

$$\therefore d\,(\text{covers } u) = -\cos u du$$

Ejer. 278. $y = \dfrac{\text{sen}^m x}{\cos^n x}$. (Williamson, 27)

$$\text{L } y = m \text{ L sen } x - n \text{ L cos } x$$

$$\frac{dy}{y} = m \frac{d \text{ sen } x}{\text{sen } x} - n \frac{d \cos x}{\cos x}$$

$$dy = \frac{\text{sen}^{m-1} x}{\cos^{n-1} x} (m \cos^2 x + n \text{ sen}^2 x)\, dx$$

9. $$d \text{ sen L tg } x = \cos \text{ L tg } x \, d \text{ L tg} x$$

$$= \cos \text{ L tg } x \frac{d \text{ tg } x}{\text{tg } x} = \cos \text{ L tg } x \frac{\sec^2 x}{\text{tg } x} dx$$

280. $$y = \frac{\sec \text{ cosec } x}{\text{tg cot } x}$$

$$\frac{dy}{dx} = \frac{\text{tg cot } x \text{ tg cosec } x \sec \text{ cosec } x + \sec \text{ cosec } x \sec^2 \cot x \cot x \text{ cosec } x}{\text{tg}^2 x \cot x}$$

* II. MÉTODO.—En Algebra Superior se desmuestran las ecuaciones de Euler, en las que las líneas trigonométricas, mediante el signo $i=\sqrt{-1}$, toman la forma exponencial:

$$\text{sen } x = \frac{e^{ix} - e^{-ix}}{2\,i}, \qquad \cos x = \frac{e^{ix} + e^{-ix}}{2}.$$

Diferenciamos la primera y resulta la segunda como derivada:

$$d \text{ sen } x = d \frac{e^{ix} - e^{-ix}}{2\,i} = \frac{1}{2\,i} (d\ e^{ix} - d\ e^{-ix})$$

$$= \frac{1}{2\,i} [\, e^{ix} dix - e^{-ix}\, d\, (-ix)\,] = \frac{1}{2\,i} (e^{ix}\, i + e^{-ix}\, i)\, dx$$

$$= \tfrac{1}{2} (e^{ix} + e^{-ix})\, dx = \cos x\, d\, x$$

$$\therefore d \text{ sen } x = \cos x\, d\, x.$$

Para encontrar las demás diferenciales trigonométricas basta reducir las lineas á sen y cos: $\cos x = \text{sen } (90^o\ x)$, $\text{tg } x = \frac{\text{sen } x}{\cos x}$, etc.

29. **Funciones circulares.**—*A*). DEFINICIONES. Para despejar á y de la función trigonométrica

$$u = \text{sen } y,$$

en la que y es el arco correspondiente al seno u, se conviene en escribir

$$y = \text{arc sen } u.$$

y se lée «arco cuyo seno es u» ó, mejor, «arco seno u».

Tal es la FUNCIÓN CIRCULAR SENO.

De los demás funciones trigonométricas,

$$u=\cos y, \quad u=\text{tg}\, y, \quad u=\sec y, \quad \text{etc.};$$

salen las funciones circulares respectivas,

$$y=\text{arc cos } u, \quad y=\text{arc tg } u, \quad y=\text{arc sec } u, \quad \text{etc.}$$

Según esto, las funciones circulares expresan el arco en términos de las líneas trigonométricas; y son, por tanto, inversas de las funciones trigonométricas.

B). VALOR NUMÉRICO DE LAS FUNCIONES CIRCULARES. — El valor conocido sen $30^\circ=\frac{1}{2}$ nos dá: $30^\circ=$arc sen $\frac{1}{2}$; y como $360^\circ=2\pi$ r o $180^\circ=\pi$, tendremos:

$$\frac{1}{6}\pi = \text{arc sen } 0{,}5 \quad \therefore \quad \pi = 6 \text{ arc sen } 0{,}5$$

Para tg $45^\circ=1$, encontramos, $45^\circ=\frac{\pi}{4}=$arc tg 1; y

$$\text{para } \cos 30^{\circ} = \tfrac{1}{2}\sqrt{3}, \quad \frac{1}{6}\pi = \text{arc cos } \tfrac{1}{2}\sqrt{3}.$$

Luego,

$$\pi = 6 \text{ arc sen } 0{,}5 = 4 \text{ arc tg } 1 = 6 \text{ arc cos } \tfrac{1}{2}\sqrt{3}.$$

C). Relaciones circulares.

a). Sea $y = \text{arc sen } u \therefore u = \text{sen } y$,

$$1 - u^2 = 1 - \text{sen}^2\, y = \cos^2 y \therefore \cos y = \sqrt{1-u^2}, \; y = \text{arc cos } \sqrt{1-u^2}.$$

Luego,

$$\text{arc sen } u = \text{arc cos } \sqrt{1-u^2}$$

Si se quiere expresar arc sen u en función de arc tg u, escribimos:

$$\text{tg } y = \frac{\text{sen } y}{\cos y} = \frac{\text{sen } y}{\sqrt{1-\text{sen}^2\, y}} = \frac{1}{\sqrt{1-u^2}} \therefore y = \text{arc tg} \frac{u}{\sqrt{1-u^2}}$$

Luego,

$$\text{arc sen } u = \text{arc tg} \frac{u}{\sqrt{1-u^2}}$$

Del mismo modo se encuentran las relaciones:

$$\text{arc sen } u = \text{arc cot}\frac{\sqrt{1-u^2}}{u} = \text{arc sec}\frac{1}{\sqrt{1-u^2}} = \text{arc cosec}\frac{1}{u}$$

3). Calculemos, ahora, la suma arc sen a+arcsen b.

Sea arc sen $a=u$ $\therefore$ $a=\text{sen } u$, $\cos u = \sqrt{1-a^2}$,

arc sen $b = v$ $\therefore$ $b = \text{sen } v$, $\cos v = \sqrt{1-b^2}$.

Sabemos que $\text{sen }(u+v) = \text{sen } u \cos v + \text{sen } v \cos u$;

luego, $u+v = \text{arc sen }(\text{sen } u \cos v + \text{sen } v \cos u)$

$$= \text{arc sen }(a\sqrt{1-b^2} + b\sqrt{1-a^2})$$

c). Calculemos, además, arc tg a + art tg b.

Sea arc tg $a=u$ $\therefore$ $a=\text{tg } u$; arc tg $b=v$ $\therefore$ $b=\text{tg } b$.

$$\text{tg }(u+v) = \frac{\text{tg } u + \text{tg } v}{1-\text{tg } u \text{ tg } v} \therefore u+v = \text{arc tg}\frac{\text{tg } u + \text{tg } v}{1-\text{tg } u \text{ tg } v}$$

o bien, $$\text{arctg } a + \text{arctg } b = \text{arctg } \frac{a+b}{1-ab}.$$

d). Hacemos $a = b$ en lás fórmulas anteriores:

$$2 \text{ arc sen } a = \text{arc sen } 2\, a \sqrt{1-a^2},$$

$$2 \text{ arc tg } a = \text{arc sen } \frac{2\,a}{1-a^2}. \quad \text{(Greenhill, 27)}$$

DIFERENCIAL DE ARCO SENO.— $y = \text{arc sen } u.$

$$u = \text{sen } y, \therefore du = \cos y \, dy, \; dy = \frac{du}{\cos y}.$$

Para eliminar a y del segundo miembro, acudimos a igualdad sen $y = u \;\therefore\; 1 - \text{sen}^2 y = 1 - u^2 = \cos^2 y$

$$\therefore \cos y = \sqrt{1-u^2}$$

$$\therefore d(\text{arc sen } u) = \frac{du}{\sqrt{1-u^2}} \qquad (15)$$

La diferencial de arco seno u es igual a la diferencial de u dividida por la raiz cuadrada de la unidad disminuida en el cuadrado de u.

Ej. 281. $d \text{ arc sen } (ax+b) = \dfrac{d(ax+b)}{\sqrt{1-(ax+b)^2}} = \dfrac{adx}{\sqrt{1-(ax+b)^2}}$

2. $d \text{ arc sen} \dfrac{1}{x} = \dfrac{d\,\dfrac{1}{x}}{\sqrt{1-\left(\dfrac{1}{x}\right)^2}} = \dfrac{-\dfrac{1}{x^2}}{\sqrt{1-\dfrac{1}{x^2}}}dx - \dfrac{dx}{x\sqrt{x^2-1}}.$

3. $d\,(\text{arc sen}\sqrt{x})^n = d\cdot\text{arc}^n \text{ sen}\sqrt{x} = n \text{ arc}^{n-1} \text{ sen}\sqrt{x}$

$$d \text{ arc sen}\sqrt{x} = n \text{ arc}^{n-1} \text{ sen}\sqrt{x}\, \frac{\frac{dx}{2\sqrt{x}}}{\sqrt{1-x}} =$$

$$n \text{ arc}^{n-1} \text{ sen}\sqrt{x} \cdot \frac{dx}{2\sqrt{x-x^2}}.$$

4. $$d\, \text{L arc sen}\, x = \frac{d \text{ arc sen}\, x}{\text{arc sen x}} = \frac{dx}{\text{arc sen x} \cdot \sqrt{1-\text{x}^2}}.$$

5. $y = x^{\text{arc see}\, x}$. $\quad \text{L}\, y = \text{arc sen}\, x\, \text{L}\, x$

$$\frac{dy}{y} = \text{arc sen}\, x\, \frac{dx}{x} + \text{L}\, x\, \frac{dx}{\sqrt{1-x^2}}$$

$$\therefore dy = x^{\text{arc sen}\, x}\left(\frac{\text{arc sen}\, x}{x} + \frac{\text{L}\, x}{\sqrt{1-x^2}}\right)dx$$

Diferencial de arco coseno. $y = \text{arc cos}\, u$.
$u = \cos y,\ du = -\text{sen}\, y\, dy = -\sqrt{1-u^2}\, dy$

$$\therefore d\,(\text{arc cos}\, u) = -\frac{du}{\sqrt{1-u^2}}$$

La diferencial de arc cos u, es igual a la de arc sen u, *con signo menos.*

$$\therefore \pm\frac{du}{\sqrt{1-u^2}} = d \text{ arc sen}\, u \quad \text{o} \quad d \text{ arc cos}\, u.$$

DIFERENCIAL DE ARCO TANGENTE. $y=\text{arc tg } u$.

$$u=\text{tg } y,\ du=\sec^2 y\, d\, y=(1+\text{tg}^2\, y)\, dy=(1+u^2)\, dy:$$

$$\therefore\ d\ (\text{arc tg } u)=\frac{du}{1+u^2} \qquad (16)$$

La diferencial de arc tg u es igual á du dividida por la unidad aumentada en el cuadrado de u.

Ej. 286. d arc tg $(a+bx+cx^2)=\dfrac{d\ (a+bx+cx^2)}{1+(a+bx+cx^2)^2}$

$$=\frac{b+2\, c\, x}{1+(a+bx+cx^2)^2}\, dx$$

7. D arc tg $\dfrac{3\,x}{1+x^2}=D\dfrac{3\,x}{1+y^2}:\left[1+\left(\dfrac{3\,x}{1-x^2}\right)^2\right]$

$$=\frac{(1+x^2)\,3-3\,x\,2\,x}{(1+x^2)^2}\ \frac{(1+x^2)^2\ 9\,x^2}{(1+x^2)^2}=\frac{3-3\,x^2}{9\,x^2\,(1+^2)x^2}=\frac{1-x^2}{3x^2(1+x^2)^2}$$

8. $y=\text{arc tg}\sqrt{\dfrac{1-\cos x}{1+\cos x}}$. (Bowser, 57)

$$\sqrt{\frac{1-\cos\ x}{1+\cos\ x}}=\sqrt{\frac{(1-\cos x)^2}{1-\cos^2 x}}=\frac{1-\cos x}{\text{sen } x}=\frac{2\ \text{sen}^2\ \frac{1}{2}\, x}{2\ \text{sen}\ \frac{1}{2}\, x\cos\frac{1}{2}\, x}$$

$$=\operatorname{tg}\tfrac{1}{2}x \therefore y=\operatorname{arc}\operatorname{tg}\operatorname{tg}\tfrac{1}{2}x \text{ ó } \operatorname{tg}\tfrac{1}{2}x=\operatorname{tg} y$$

$$\text{ó } y=\tfrac{1}{2}x \therefore dy=\tfrac{1}{2}d\,x.$$

9. $y=\mathbf{L}\operatorname{arctg} u.\quad d\,y=\dfrac{d\operatorname{arct} u}{\operatorname{arc}\operatorname{tg} u}$

$$=\frac{du}{(1+u^2)\operatorname{arctg} u}.$$

290. $y=(\operatorname{sen} x)^{\operatorname{arctg} u}.\quad \mathbf{L}\,y=\operatorname{arctg} u\,\mathbf{L}\operatorname{sen} x.$

$$\frac{d\,y}{y}=\operatorname{arct} g\,u\,\frac{\cos x}{\operatorname{sen} x}\,d\,x+\mathbf{L}\operatorname{sen} x\,\frac{du}{1+v^2}$$

$$\therefore dy=\operatorname{sen} x)^{\operatorname{arctg} u}\left(\operatorname{arctg} u.\cot x\,d\,x+\frac{\mathbf{L}\operatorname{sen} x\,du}{1+u^2}\right)$$

Diferencial de arc cot u. $y=\operatorname{arc}\cot u$.

$$u=\cot y,\ du=-\operatorname{cosec}^2 y\,dy=-(1+\cot^2 y)\,dy$$

$$\therefore d\operatorname{arc}\cot u=-\frac{du}{1+u^2}$$

Las diferenciales de arc tg y arc cot son iguales con signos contrarios.

Diferencial de arc sec u. $y=\operatorname{arc}\sec u$

$$u=\sec y\ du=\operatorname{tg} y\sec y\,dy=\sqrt{\sec^2 y-1}\,\sec y\,dy$$

$$\therefore d\operatorname{arc}\sec u=\frac{du}{u\sqrt{u^2-1}}$$

DIFERENCIAL DE ARC COSEC u. $y = \text{arc cosec } u$

$$u = \text{cosec } y, \; du = -\cot y \text{ cosec } y \, dy = -u\sqrt{1-u^2}\, dy$$

$$\therefore d \text{ arc cosec } u = -\frac{du}{u\sqrt{u^2-1}}$$

DIFERENCIA DE ARC VERSENO. $y = \text{arc vers } u$

$$u = \text{vers } y\,;\; du = \text{sen } y\, dy = \sqrt{2u - u^2}\, dy$$

$$\therefore d \text{ arc vers } u = \frac{du}{\sqrt{2u-u^2}}$$

DIFERENCIAL DE ARC COVERSENO. $y = \text{arc covers } u$

$$u = \text{covers } y\,,\; du = -\cos y\, dy = -\sqrt{2u-u^2}\, dy$$

$$\therefore d \text{ arc covers } u = -\frac{du}{\sqrt{2u-u^2}}$$

Ejer. 291. $y = \text{arc sen}\dfrac{x}{\sqrt{1+x^2}}$ (Bowser 55).

$$dy = d\frac{x}{\sqrt{1+x^2}} : \sqrt{1-\frac{x^2}{1+x^2}} = \frac{1}{\sqrt{(1+x^2)^3}} : \frac{1}{\sqrt{1+x^2}} = \frac{dx}{1+x^2}$$

2. $y = \text{arc } \cos\sqrt{1-x^2}.\; dy = -\dfrac{d\sqrt{1-x^2}}{\sqrt{1-(1-x^2)}} = \dfrac{dx}{\sqrt{1-x^2}}.$

En efecto, $\cos y = \sqrt{1-x^2} \therefore 1 - \cos^2 y = x^4, \therefore y = \text{arcren } x.$

3. $y = \text{arc sec}\dfrac{1}{2x^2 1}.$ (Todhunter, 55)

$$d \operatorname{arc} \sec u = \frac{du}{u\sqrt{u^2-1}} \therefore du = -\frac{4\,x\,d\,x}{(2\,x^2-1)^2},$$

$$\sqrt{u^2-1} = \sqrt{\left(\frac{1}{2\,x^2-1}\right)^2-1} = \sqrt{\frac{1-(2\,x^2-1)^2}{(2\,x^2-1)^2}} = \frac{2\,x\sqrt{x^2+1}}{d\,x^2-1}$$

$$dy = -\frac{4\,x\,d\,x}{(2\,x^2-1)^2} : \frac{2\,x\sqrt{x^2+1}}{(2\,x^2-1)^2} = -\frac{2}{\sqrt{1-x^2}}\,dx$$

4. $y = \operatorname{tg} x \operatorname{arc} \operatorname{tg} x \therefore \frac{dy}{dx} = \frac{\operatorname{tg} x}{1+x^2} + \operatorname{arctg} x \sec^2 x.$

5. $y = \mathbf{L}\,(\operatorname{arc} \cos \sqrt{1+x^2})$ (Sturm, 68)

$$y = \mathbf{L} \operatorname{arc} \operatorname{sen} x = \frac{dx}{\sqrt{1-x^2} \operatorname{arc} \operatorname{sen} x}$$

6. $D\,x\,e^{\operatorname{arc} \operatorname{tg} x} = e^{\operatorname{arc} \operatorname{tg} x}\,d \operatorname{arc} \operatorname{tg} x = \frac{e^{\operatorname{arc} \operatorname{tg} x}}{1+x^2}\,dx$

7. $d \operatorname{sen} \operatorname{arc} \operatorname{sen} x = \frac{\cos \operatorname{arc} \operatorname{sen} x}{\sqrt{1-x^2}}\,dx$

8. $d \operatorname{arc} \operatorname{sen} \sqrt{\operatorname{sen} x}\ \frac{d\sqrt{\operatorname{sen} x}}{\sqrt{1-\operatorname{sen} x}}\ \frac{\cos x}{2\sqrt{\operatorname{sen} x - \operatorname{sen}^2 x}}\,dx$

$$= \tfrac{1}{2}\sqrt{\frac{1-\operatorname{sen}^2 x}{\operatorname{sen} x\,(1-\operatorname{sen} x)}}\,dx = \tfrac{1}{2}\sqrt{\frac{1+\operatorname{sen} x}{\operatorname{sen} x}}\,d\,x = \tfrac{1}{2}\sqrt{1+\operatorname{cosec} x}\,dx$$

9. d arc^n sen arc^m cos x

$$= -\text{arc}^n \text{ sen } x \, . \, m \text{ ar}x^{m-1} \cos x \frac{dx}{\sqrt{1-x}}$$

$$+ \text{arc}^m \cos x \, n \, \text{arc}^{n-1} \text{ sen } x \frac{dx}{\sqrt{1-x^2}}$$

$$= \frac{\text{arc}^{u-1} \text{ sen } x \text{ arc cos}^{m-1} x}{\sqrt{1-x^2}} (u \text{ arc}^m \cos x \, m \text{ arc}^u \text{ sen o}) \, dx.$$

* 300. $u = \text{arc tg} \dfrac{5x - 10x^3 + x^5}{1 - 10x^2 + 5x^4}$ (PEACOCK, 39)

$$du = d \text{ arc tg } \frac{v}{t} = \frac{a \frac{v}{t}}{1 + \left(\frac{v}{t}\right)^2} = \frac{tdv - vdt}{v^2 + t^2}.$$

$$tdv = 5 (1 - 10x^2 + 5x^4)(1 - 6x + x^4)$$

$$vdi = 20x^2 (x^2 - 1)(5 - 10x^2 + x^4)$$

$$\therefore du = \frac{5}{1 + x^2} dx$$

30. Formulario diferencial.

FUNCIONES ALGEBRAICAS

1. Término constante $d\,(a) = 0$
2. Función logarítmica $d\,(\text{L}\, u) = \dfrac{d\,u}{u}$
3. Suma de funciones $d\,(u + v) = du + dv$
4. Signo y coeficiente $d\,(\pm au) = \pm adu$

5. Potencia	$d\ (u^n) = nu^{n-1}\, du$
6. Inversa	$d\left(\frac{1}{u}\right) = -\frac{du}{u^2}$
7. Raíz cuadrada	$d\ (\sqrt{u}) = \frac{du}{2\sqrt{u}}$
8. Producto de funciones	$d\ (uv) = u\,dv + v\,du$
9. Cuociente de funciones	$d\left(\frac{u}{v}\right) = \frac{v du - u dv}{v^2}$

FUNCIONES TRASCENDENTES

10.	Función exponencial	$d\ (a^u) = a^u \mathrm{L}\, a\, du$
11.	» »	$d\ (e^x) = e^x dx$
12.	» »	$d\ (\mathrm{sen}\ u) = \cos u\ du$
13.	» coseno	$d\ (\cos u) = -\mathrm{sen}\ u du$
14.	» tangente	$d\ (\mathrm{tg}\ u) = \sec^2 u\ du$
15.	» arco seno	$d\ (\mathrm{arc\ sen}\ u) = \frac{du}{\sqrt{1-u^2}}$
16.	» arco tangente	$d\ (\mathrm{arc\ tg}\ u) = \frac{du}{1+u^2}$

31. EJERCICIOS. PRIMERA SERIE.

301. $y = \frac{e^x - e^{-x}}{e^x = e^{-x}}$. (CARR, 263)

Cuociente de funciones exponenciales:

$$dy = \frac{(e^x + e^{-x})\,(e^x + e^{-x}) - (e^x - e^{-x})\,(e^x - e^{-x})}{(e^x + e^{-x})^2}\, dx$$

$$= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})}{(e^x + e^{-x})^2}\, dx = \frac{4}{(e^x + e^{-x})^2}\, dx$$

2. $y = ab^{x^x} = ab^{x^x}$ (STURM, 59)

Función excepcional. Hacemos $u = x^x$ y aplicamos logaritmos:

$$Ly = La + uLb \therefore dy = Lbdu.$$

Aplicamos de nuevo L, $Lu = xLx$

$$\therefore \frac{du}{u} = \frac{dx}{x} + Lxdx \therefore du = x^x(1 + Lx)\,dx$$

$$\therefore dy = ab^{x^x}\, x^x\,(1+Lx)\,dx.$$

3. $u = L\{L[\,\ldots L(Lx)\,]\} = L_n x.$ (GRÉGORY, 2)

Sabemos que $dLu = \frac{du}{u}$; y si $u = Lx$:

$$Ld\,(Lx) = dL_2x = \frac{dLx}{Lx} = \frac{dx}{xLd}.$$

Sea, ahora, $u = dL_2x$:

$$dL\,(L_2x) = d\,L_3x = \frac{dLx}{Lx} = \frac{dx}{xLxL_2x}.$$

Sea $u = L_3x$:

$$dL\,(L_3x) = dL_4x = \frac{dL_3x}{L_3x} = \frac{dx}{xLxL_2xL_3x}$$

$$\therefore dL^nx = \frac{dx}{xLxL_2x\ldots L_{n-1}x}.$$

4. $d\,(Ae^{mx} + Be^{-mx})$ (COMBEROUSSE, 490)

$$= (Ae^{mx}.\,m - Be^{-mx}m)\,dx = m\,(Ae^{mx} - B^{-mx})\,dx$$

5. $$de^{e^{e^x}} = de^u = e^u de^{e^x} = e^u dv = e^u e_v de^x$$

$$= e^{e^{e^x}} e^{e^x} e^x dx$$

6. $$d\,\mathrm{L}\,\frac{a+bx}{a-bx} = d[\,\mathrm{L}\,(a+bx) - \mathrm{L}\,(o-bx)\,]$$

$$= \left(\frac{b}{a+bx} + \frac{b}{a-bx}\right) dx = \frac{2ab}{a^2 - b^2x^6}\,dx.$$

7. $$d\,\mathrm{L}\,\sqrt{\frac{1+x}{1-x}} = \tfrac{1}{2}\,d[\,\mathrm{L}(1-x) - \mathrm{L}(1+x)\,]$$

$$= \tfrac{1}{2}\left(-\frac{1}{1-x} - \frac{1}{1+x}\right) dx = \frac{dx}{x^2-1}$$

8. $$d\,\mathrm{L}\,[\,(x-a)^m (x-b)^n (x-c)^p \ldots\,]$$

$$= d\,[\,m\mathrm{L}\,(x-a) + n\mathrm{L}\,(x-b) + p\,\mathrm{L}\,(x-c) \ldots\,]$$

$$= \left(\frac{x\,m}{x-a} + \frac{n}{x-b} + \frac{p}{x-c} + \ldots\right) dx$$

9. $$d\,\mathrm{L}\,(x + \sqrt{x^2 - a^2}) + \operatorname{arc\,sec} \frac{x}{a}$$ (ID. 511

$$\mathrm{dy} = \frac{d(x + \sqrt{x^2 - a^2})}{x + \sqrt{x^2 - a^2})} + d\,\frac{x}{a} : \frac{x}{a}\sqrt{\frac{x^2}{a^2} - 1}$$

$$= \left[\frac{\sqrt{x^2 - a^2} + x}{(x + \sqrt{x^2 - a^2})\,\sqrt{x^2 - a^2}} + \frac{a}{x\sqrt{x^2 - a^2}}\right] dx = \frac{1}{x}\sqrt{\frac{x+a}{x-a}}$$

310. $$d \text{ arc tg } \frac{a+x}{1-ax} = d\, \frac{a+x}{1=ax} : \left[1+\left(\frac{a+x}{1-ax}\right)\right]^2$$

$$= \frac{(1-ax)+(a+x)a}{(1-ax)^2} \cdot \frac{(1-ax)^2}{(1-ax)^2+(a+x)^2}\, dx = \frac{1}{1+x^2}\, ax$$

(*Continuará*)